Notes: The solution drawn on the map above is only one possible solution to Challenge #1 and assumes there is only one bus in the fleet. It is not necessarily the best (most energy or time-efficient) solution. Below is one possible way students might calculate distance and time for this particular solution (all distances have been measured using the scale bar):

- Distance from Depot to Stop 1: ~1.36 mi
- Distance from Stop 1 to Stop 2: ~1.03 mi
- Distance from Stop 2 to Stop 3: ~0.81 mi
- Distance from Stop 3 to Stop 4: ~0.88 mi
- Distance from Stop 4 to Stop 5: ~1.13 mi
- Distance from Stop 5 to Stop 6: ~0.88 mi
- Distance from Stop 6 to Stop 7: ~0.88 mi
- Distance from Stop 7 to Depot: ~1.8 mi

**Total distance:** ~8.77 miles

**Time of complete circuit:**

\[
(8.77 \text{ miles} \div 0.5 \text{ mile/minute}) + (1 \text{ minute/stop} \times 7 \text{ stops}) = -24.5 \text{ minutes}
\]
Building Better Buses: Transportation Design Challenges
Teacher Solutions: Challenge #2

1. Use ratios to solve:
Which is more fuel efficient: a bus powered by E85 biofuel or a bus powered by regular gasoline?

**Bus 1: E85**
- Distance driven = 108 miles
- Fuel used = 36 gal ÷ 2 = 18 gal
- Fuel efficiency = miles per gallon
  = 108 mi ÷ 18 gal
  = 6 mi/gal

**Bus 2: Regular gasoline**
- Distance driven = 243 miles
- Fuel used = 36 gal x (3/4) = 27 gal
- Fuel efficiency = miles per gallon
  = 243 mi ÷ 27 gal
  = 9 mi/gal

**A bus running on regular gasoline is more fuel efficient compared to a bus running on E85 biofuel.**

2. Use ratios and cross-multiplying to solve:
How the carbon dioxide produced by a bus burning regular gasoline compares to the carbon dioxide produced by a bus running on E85 biofuel for the same distance driven.

- First, decide on a distance. Here, we’re going to choose 6 miles, the distance a bus can travel on 1 gallon of E85 biofuel. This makes it easy to calculate how many kilograms of CO₂ are produced by a bus running on E85:

  Fuel efficiency of a bus running on E85 = 6 mi/1 gal

  \[
  \frac{10 \text{ gal E85}}{12 \text{ kg CO}_2} = \frac{1 \text{ gal E85}}{? \text{ kg CO}_2}
  \]

  \[
  12 \text{ kg CO}_2 \times 1 \text{ gal E85} ÷ 10 \text{ gal E85} = 1.2 \text{ kg CO}_2
  \]

- There are several ways to solve for how many kilograms of CO₂ are produced by a bus traveling 6 miles on regular gasoline. One way is to use the same method as above to solve for the CO₂ produced by a bus traveling 9 miles (1 gallon) on regular gasoline, then divide the answer by 1.5 since 9 miles is 1.5 times longer than 6 miles:

  Fuel efficiency of a bus running on regular gasoline= 9 mi/1 gal

  \[
  \frac{5 \text{ gal gas}}{45 \text{ kg CO}_2} = \frac{1 \text{ gal gas}}{? \text{ kg CO}_2}
  \]

  \[
  45 \text{ kg CO}_2 \times 1 \text{ gal gas} ÷ 5 \text{ gal gas} ÷ 1.5 = 6 \text{ kg CO}_2
  \]

**Regular gasoline releases 5 times as much CO₂ as E85 for the same distance traveled!**
3. Use ratios and canceling out units to solve:
How the cost of running buses on E85 biofuel compares to running buses on regular gasoline for the same distance.

- Let’s pick 9 miles for our distance. We know it will cost $X for a bus running on regular gasoline to go 9 miles, since this is the distance the bus can travel on 1 gallon of gasoline, and gas costs $X/gal.

- If a bus needs 1 gallon of E85 to drive 6 miles, then it needs 1.5 gallons of E85 to drive 9 miles since 9 is 1.5 times larger than 6:

\[
\frac{6 \text{ miles}}{1 \text{ gallon E85}} = \frac{9 \text{ miles}}{1.5 \text{ gallons E85}}
\]

- Therefore, we can just multiply the price per gallon of E85, $Z, by 1.5 and compare our result to the price per gallon of regular gasoline, $X:

\[
\frac{Z}{1 \text{ gallon E85}} \times 1.5 \text{ gallons} = Y
\]

for a bus to travel 9 miles on E85 compared to $X on gasoline.

Building Better Buses: Transportation Design Challenges
Teacher Solutions: Challenge #3

1. Use a variety of methods to:
   a. Minimize CO₂ emissions produced by your bus system.
   b. Minimize the cost of your plan.

- For this Challenge, students can use very similar methods to optimize their bus system plan as they did in Challenges #1 & 2. Encourage students to draw pictures or tables, pay attention to their units, and use dimensional analysis to help them figure out their calculations.

- Students might find it confusing to do calculations for 50/50 electricity since they might not understand how to think about it. You can give them an example, such as how to figure out how many pounds of coal are needed for a bus to drive 10 miles on 50/50 electricity. In this example, students can think of the bus as traveling 5 miles using 100% coal (which has a cost and produces CO₂) and 5 miles on wind power (which is free and doesn’t produce CO₂).

- Instead of grading students on how well they were able to optimize their solutions, focus on the methodologies students used to come up with their solutions. Students should be able to explain their thinking as well as their problem solving strategies. There is no one correct answer to this Challenge.
• If your students struggle with ratios, encourage them to draw the ratio as a picture (e.g., by representing each variable with a symbol). Alternatively, you can give your students physical items to use to represent the variables and create ratios, like marbles, coins, or pieces of candy.

• The specific numbers reported in the solutions above may not be the same numbers your students come up with, depending on the conditions they choose to start with (e.g., how many miles they choose as the distance their buses travel).

• Give your students plenty of extra scratch paper so that they have a lot of room for problem solving and for making mistakes.

• Need more examples of math and computational problem solving and modeling? Check out these additional resources:
  
  ▪ **Mathematics Assessment Project: Classroom Challenges**
  The Mathematics Assessment Project aims to bring the Common Core State Standards (CCSSM) to life in a way that will help teachers and their students turn their aspirations for achieving them into classroom realities.

  ▪ **California Mathematics Project: K-8 Modeling Resources**
  The California Mathematics Project (CMP) is a K-16 network dedicated to providing students a rich, rigorous, and coherent mathematics curriculum taught by competent and confident mathematics teachers who foster ALL students’ proficiency in mathematics—achieving equity in quality. CMP enhances teachers’ mathematical content knowledge and pedagogical content knowledge that is aligned to the California Mathematics Standards and Framework.